



UNIT 8

THE AMORTIZATION SCHEDULE

Most people who own or want to purchase a home take out a **mortgage**—a long-term lending arrangement that requires the borrower to make periodic payments to the lender. Most mortgages will require monthly payments that typically extend over a period of 20 to 40 years, making the mortgage the largest financial obligation an individual is ever likely to undertake. As a result, it is important for the homeowner to understand the details of the obligation, including the way in which the lender uses the payments.

To illustrate, most mortgages are **amortized**, which means that the payments are structured in such a way as to make interest payments on the outstanding balance while simultaneously repaying the amount borrowed over a set period of time. Because most mortgages are so long, however, most homeowners usually repay the lender two or three times the amount of the money they borrowed in the first place. The purpose of this unit is to explain how this happens, and to examine ways to lessen the burden of the mortgage.

The focus of this unit is the **amortization schedule**, a tabular listing that shows the size of the periodic payment, the division of the payment between principal and interest, and the amount still owed. The amortization schedule can be used for both the **fixed-rate mortgage** which has payments of equal size over the duration of the mortgage, and the **variable rate mortgage** which has payments that change as interest rates change.

Student Learning Objectives:

- Students will discover how mortgage payments are allocated to principal and interest.
- Students will learn how to construct and interpret an amortization schedule.
- Students will discover how to reduce the burden of the mortgage.

Key Concepts:

amortized
interest

amortization schedule
principal

fixed-rate mortgage
variable-rate mortgage

LESSON 8-1: INTEREST AND THE REDUCTION OF PRINCIPAL

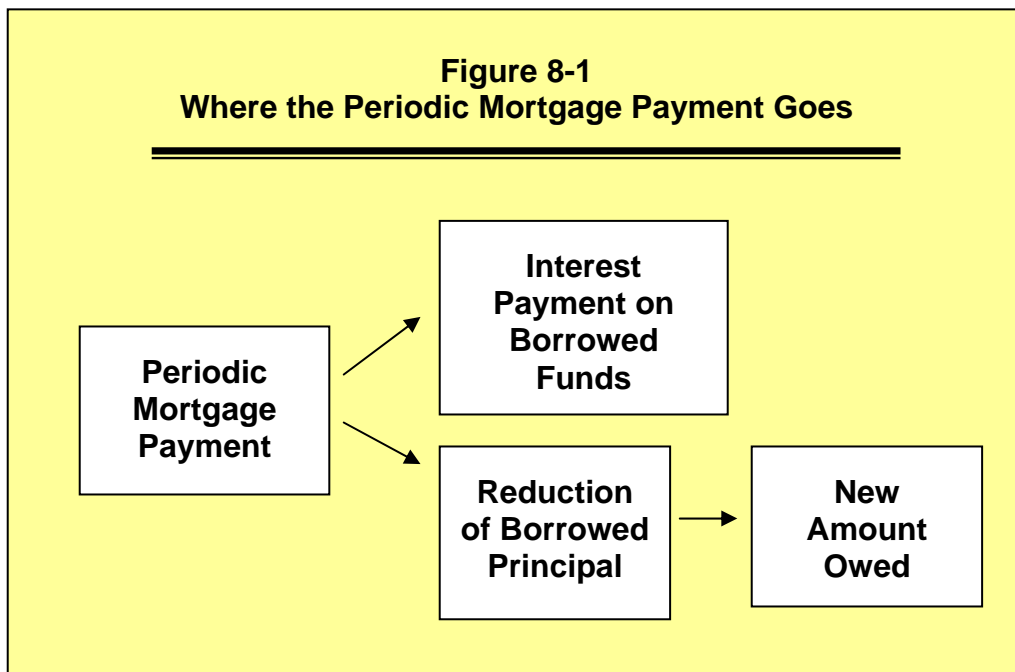
There are several steps involved when constructing an amortization schedule. Fortunately, the most difficult step, that of finding the size of the monthly payment, was already covered in Lesson 6-1 on page zzz. As a result, the focus of this lesson is to explain how the payment is separated into two parts: (1) interest on the amount of principal still owed, and (2) reduction of the principal. The key thing to remember is that the reduction of principal is a residual, with interest payments being paid first.

Materials Needed:

- At least one copy of *Activity Sheet 8-1: Interest and the Reduction of Principal* for each student.
- *Figure 8-1* transparency or PowerPoint presentation file.

Teaching the Lesson:

1. Start with an introduction of the lesson by showing *Figure 8-1*. The key point to emphasize is that the payment itself is divided into two parts, interest on borrowed funds and reduction of borrowed principal.



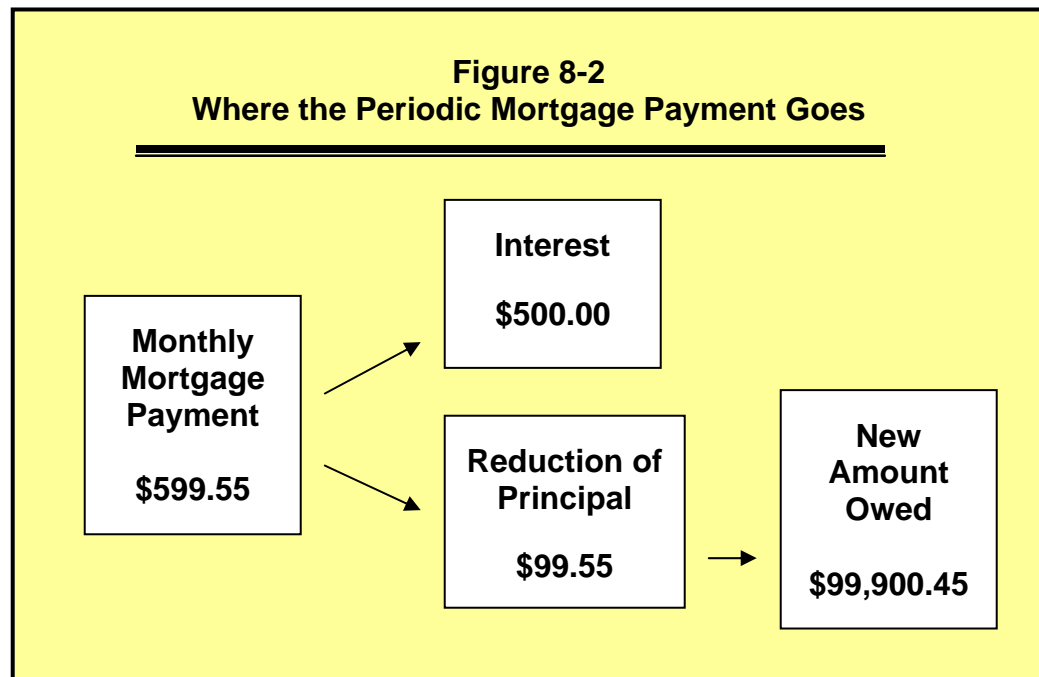
This is a repetitive process, with every periodic (usually monthly) payment reducing the amount owed—which in turn reduces the amount of interest paid when the next payment is made.

- In order to find out how much interest is owed, we use the simple interest formula $I=PRT$, and then substitute the values to find the interest. The numbers used here represent the interest computation for a \$100,000, 6-percent, 30-year mortgage (.083333 is the decimal equivalent of $1/12^{\text{th}}$ of a year).

$$\begin{aligned} \text{Interest} &= (\text{Principal})(\text{Rate})(\text{Time}) \\ &= (\$100,000)(6\%)(1 \text{ month}) \\ &= (\$100,000)(.06)(.083333) = \$500 \end{aligned}$$

- The 100,000, 6-percent, 30-year mortgage is the same as the one computed in *Figure 6-1* on page zz (you may want to show *Figure 6-1* first as a reminder of how the monthly payment is determined). Because the size of the mortgage payment is already known, all we do is subtract the interest from the payment to find the reduction of principal. This, in turn, leads to a smaller principal.

Figure 8-2 below illustrates the process. We start with the \$599.55 payment and then subtract \$500 which is the monthly interest on the principal. The residual, \$99.55, is then used to reduce the principal to a new (and lower) value of \$99,900.45.



- The next step is to repeat the process, only this time the amount of interest owed goes down slightly because less principal is owed:

$$\begin{aligned} \text{Interest} &= (\text{Principal})(\text{Rate})(\text{Time}) \\ &= (\$99,900.45)(.06)(.083333) = \$499.50 \end{aligned}$$

5. Now that the students have seen how to use the simple interest formula to compute the periodic interest, distribute *Activity Sheet 8-1: Interest and the Reduction of Principal* which reviews the concepts reviewed in this lesson.

6. The answers for the worksheet are as follows (the \$733.76 monthly payment is from the table on page zzz):

$$(1) \text{ Interest} = (\text{Principal})(\text{Rate})(\text{Time})$$

$$= (\$100,000)(.08)(.083333) = \$666.66$$

$$\text{Principal reduction} = \$733.76 - \$666.66 = \$67.10$$

$$\text{Principal owed after one payment} = \$100,000 - \$67.10 = \$99,932.90$$

$$(2) \text{ Interest} = (\text{Principal})(\text{Rate})(\text{Time})$$

$$= (\$99,932.90)(.08)(.083333) = \$666.22$$

$$\text{Principal reduction} = \$733.76 - \$666.22 = \$67.54$$

$$\text{Principal owed after two payments} = \$99,932.90 - \$67.54 = \$99,865.36$$

$$(3) \text{ Interest} = (\text{Principal})(\text{Rate})(\text{Time})$$

$$= (\$99,865.36)(.08)(.083333) = \$665.77$$

$$\text{Principal reduction} = \$733.76 - 665.77 = \$67.99$$

$$\text{Principal owed after three payments} = \$99,865.36 - 67.99 = \$99,797.37$$

Debriefing the Lesson

In order to find the proper answer, students must deduct the residual payment (the payment after the interest is subtracted) to find the new amount owed. Intuitively, students should realize that the amount of interest owed should go down every month because the amount owed is slightly less. This amount goes down slowly at first, but then goes down faster in the later years of the mortgage.

LESSON 8-2: CONSTRUCTING THE AMORTIZATION SCHEDULE

The three problems in the previous lesson were somewhat repetitive in that the amount of interest is always computed using the amount currently owed. In fact, we could do these computations for every month of the 30-year (360 month) mortgage, and the process would not change.

Another way to deal with the matter is to summarize the monthly interest and principal reduction in the amortization table shown in *Activity Sheet 8-2*. This table does not introduce any new information, it simply helps organize the information we have already computed above.

✂ Materials Needed:

- One copy of *Activity Sheet 8-2: The Mortgage Amortization Schedule*, for each student.

✍ Teaching the Lesson:

1. The amortization schedule in *Activity Sheet 8-2* is for a 30-year, 8-percent, fixed rate mortgage of \$100,000. By fixed rate we mean that the size of the payment never changes (Lesson 8-4 deals with a variable rate mortgage).
2. Hand out the activity sheet and let the students fill in as much of the data as they can. The process is exactly the same as in the previous lesson, so little instruction should be needed.
3. The answers for the activity sheet are as follows:

The Mortgage Amortization Schedule

Payment Number	Monthly Payment	Interest Owed	Principal Reduction	New Principal Owed
1	\$733.76	\$666.66	\$67.10	\$99,932.90
2	733.76	666.22	67.54	99,865.36
3	733.76	665.77	67.99	99,797.37
4	733.76	665.32	68.44	99,728.92
5	733.76	664.86	68.90	99,660.02
6	733.76	664.40	69.36	99,590.66
7	733.76	663.94	69.82	99,520.84
8	733.76	663.47	70.29	99,450.55
9	733.76	663.00	70.76	99,379.80
10	733.76	662.53	71.23	99,308.57
11	733.76	662.06	71.70	99,236.86
12	733.76	661.58	72.18	99,164.68
13	733.76	661.10	72.66	99,092.02
14	733.76	660.61	73.15	99,018.87

☑ Debriefing the Lesson

The amortization schedule in the lesson is relatively simple in the sense that the interest is always computed on the basis of the principal owed. The table is more difficult in the sense that the same computations are done over and over again, which is why they are usually generated with the help of a computer. Borrowers can always ask for an

amortization schedule, and lenders will supply them, but they will not usually supply them unless specifically asked to do so.

One of the advantages of having an amortization schedule is that you always know exactly how much you still owe. All you have to do is to keep track of the number of payments you have already made, and then examine the schedule to see how much you still owe.

Another advantage is that it can help the homeowner determine how much interest was paid during the year for tax purposes (interest on a home mortgage is tax deductible). For example, if the first twelve payments in the amortization schedule fall in the same calendar year, the homeowner would have paid \$7,969.81 (or \$666.66 + \$666.22 + \$665.77 + ... + \$661.58) in interest that year.

Finally, you want to tell students that the complete amortization schedule will be 10 or 12 pages long since it has to account for each of the 360 payments shown in the first column. It should also be evident that most of the early monthly payments go to interest on the loan.

LESSON 8-3: MAKING EXTRA PAYMENTS

An amortized payment is one that is designed to pay interest on outstanding principal while simultaneously reducing principal over the course of the loan. In the case examined so far, a monthly payment of \$733.76 will completely pay off (or amortize) a \$100,000, 8-percent mortgage in exactly 360 months. The \$733.76 is the minimum payment, however, as most lenders allow you to make extra payments on the principal.

The advantage of making extra payments is that the interest charges will be less, and the mortgage will be paid off before the 30 years are up. In this lesson we are going to examine the consequences of making a slightly larger payments. The computations will be the same, only the results will be different, lessening the amount of money the borrower repays the lender.

Materials Needed:

- One copy of *Activity Sheet 8-3: Reducing the Interest Burden – Making Extra Periodic Payments*, for each student.

Teaching the Lesson:

1. The amortization schedule in *Activity Sheet 8-3* is for a 30-year, 8-percent, fixed rate mortgage of \$100,000. By fixed rate we mean that the size of the payment never changes (the next activity will deal with a variable rate mortgage).

2. The minimum required payment for this mortgage is the same as before, or \$733.76. Rather than pay that amount, however, the borrower is going to pay an extra \$100 every month in an attempt to reduce the burden of the interest and pay off the loan a little bit early. *(Intuitively, this should make sense to the students: if you pay less interest to the lender over the course of the loan, the loan should be paid off earlier.)*
3. While this may seem to complicate matters, the computation of monthly interest is the same, and the residual is used to reduce the principal owed. The result is that the loan is paid off somewhat earlier.
4. Have the students fill in the blanks, just as the did for the previous lesson. The answers are shown below:

The Mortgage Amortization Schedule

Payment Number	Monthly Payment	Interest Owed	Principal Reduction	New Principal Owed
1	\$833.76	\$666.66	\$167.10	\$99,832.90
2	\$833.76	665.55	\$168.21	99,664.69
3	\$833.76	664.43	\$169.33	99,495.36
4	\$833.76	663.30	\$170.46	99,324.90
5	\$833.76	662.16	\$171.60	99,153.31
6	\$833.76	661.02	\$172.74	98,980.57
7	\$833.76	659.87	\$173.89	98,806.67
8	\$833.76	658.71	\$175.05	98,631.62
9	\$833.76	657.54	\$176.22	98,455.40
10	\$833.76	656.37	\$177.39	98,278.01
11	\$833.76	655.18	\$178.58	98,099.43
12	\$833.76	653.99	\$179.77	97,919.67
13	\$833.76	652.80	\$180.96	97,738.70
14	\$833.76	651.59	\$182.17	97,556.53

5. A monthly payment of \$833.76 may not seem to make much of a difference in the first few months of the mortgage, but the effects are greater later on—eventually shortening the mortgage to 242 months, or 20.17 years.

While this may be beyond the scope of students who do not have access to a financial calculator, this computation is made with the use of the equation in Figure 6-1 on page 75zz. Rather than solve for the monthly payment, however, we enter the monthly payment of \$833.76 as a known variable and then solve for n , which is the number of months in the mortgage. The answer is (approximately) 242 months.

Debriefing the Lesson

Whenever someone makes an extra payment, or one larger than the minimum amortized payment required by the lender, the lender pays less interest over the life of the loan. Even if the borrower made only one extra payment of \$100, the principal owed will be \$100 smaller over the life of the loan. Not having to pay monthly interest on this amount for 360 months amounts to some interest savings. If an extra payment is made on a regular basis, the total savings are considerable—as shown by the \$62,384 interest savings in this lesson.

LESSON 8-4: VARIABLE RATE MORTGAGES

A popular type of mortgage is the variable rate mortgage, a mortgage whose periodic payment is subject to change over time. This is one of the most difficult types of mortgages to understand, although we have already done all of the computations in earlier lessons. Even so, this is a difficult activity and one best suited for advanced students.

Variable rate mortgages get their name from the fact that payments are recomputed periodically when interest rates change. For all practical purposes, they could be called variable *payment* mortgages, but lenders hesitate to do so because it introduces more uncertainty to the borrower. Variable rate mortgages are popular with lenders, because the cost of higher interest rates (or the benefits of lower ones) are shifted back to the borrower.

Materials Needed:

- One copy of *Activity Sheet 8-4: Variable Rate Mortgages*, for each student.

Teaching the Lesson:

- To begin, tell students that they are applying for a \$100,000, 9-percent, 25-year mortgage. The terms of the mortgage are such that:
 - a. The lender will recomputed the size of the monthly payment every six months, depending on the level of interest rates in the economy. In other words, if rates go up, the monthly payment will go up. Or, if interest rates go down, the monthly payment goes down.
 - b. In addition, the interest rate will never go up or down more than two percentage points at a time, and will never go up or down more than six percentage points over the life of the loan. (*In practice, these adjustments are made every year or two, it is only shortened here simply to facilitate the explanation.*)

- Hand out a copy of *Activity Sheet 8-4: Variable Rate Mortgages* to every student in class.
- Since the size of the monthly payment is not given, students will have to use the equation in *Figure 6-1* on page 75zz (and reproduced below) to find the amortized payment. (*The solution is:*

$$\begin{aligned} \text{Monthly Payment} &= \frac{\text{Principal}}{\text{PVIAF}} \\ &= \frac{\$100,000}{\left[\frac{1 - \frac{1}{(1+r)^n}}{r} \right]} = \frac{\$100,000}{\left[\frac{1 - \frac{1}{(1+0.0075)^{300}}}{0.0075} \right]} = \frac{\$100,000}{119.1616} = \$839.20 \end{aligned}$$

Also, note that the annual interest is 9 percent paid monthly, or .09/12 = .0075. Likewise, there are 300 months, or 25 years times 12, in the mortgage.)

- Now that we have the monthly payment, we can fill in the first six rows of the amortization schedule in the activity sheet. The reason we can't go any further is that we have to wait for the bank to tell us if the payment will change depending on changes in the interest rate.
- Assume now that we are ready to make the seventh payment, but that we have been notified that interest rates have gone up to 10 percent. This means that our new monthly payment will be computed as:

$$\begin{aligned} &= \frac{\$99,454.66}{\left[\frac{1 - \frac{1}{(1+r)^n}}{r} \right]} = \frac{\$99,454.66}{\left[\frac{1 - \frac{1}{(1+0.0083333)^{294}}}{0.0083333} \right]} = \frac{\$99,454.66}{109.5394} = \$907.93 \end{aligned}$$

The new computation reflects three changes: (1) Because 6 months have already gone by, there are 294 payments left to make. (2) The new monthly interest rate is 10 percent divided by twelve, or .10/12=.0083333. (3) According to the sixth row in the amortization schedule, we still owe \$99,454.66, so we have to change the numerator of the equation.

- Now that we have the new monthly payment, we can enter it into rows 7 through 12 in our amortization schedule. We won't go any further, however, because the payment is likely to change again if interest rates change—so have the students complete the blank spaces in these rows.

- Finally, assume that interest rates drop to 8 percent when it comes time to make the 13th and 14th payments. Because you now owe \$98,969.81 after having made twelve payments, and because you are obligated to pay off the loan in 29 more years, the new monthly payment will be:

$$= \frac{\$98,969.81}{\left[\frac{1 - \frac{1}{(1+r)^n}}{r} \right]} = \frac{\$98,969.81}{\left[\frac{1 - \frac{1}{(1+0.0066667)^{288}}}{0.0066667} \right]} = \frac{\$98,969.81}{127.8680} = \$774.00$$

Also, don't forget that the new interest rate is 8 percent divided by 12 for a decimal equivalent of 0.0066667:

- The answers to the activity sheet are as follows:

The Mortgage Amortization Schedule

Payment Number	Monthly Payment	Interest Owed	Principal Reduction	New Principal Owed
1	\$839.20	\$750.00	\$89.20	\$99,910.80
2	\$839.20	749.33	\$89.87	99,820.93
3	\$839.20	748.66	\$90.54	99,730.38
4	\$839.20	747.98	\$91.22	99,639.16
5	\$839.20	747.29	\$91.91	99,547.26
6	\$839.20	746.60	\$92.60	99,454.66
7	\$907.93	828.79	\$79.14	99,375.52
8	\$907.93	828.13	\$79.80	99,295.72
9	\$907.93	827.46	\$80.47	99,215.25
10	\$907.93	826.79	\$81.14	99,134.11
11	\$907.93	826.12	\$81.81	99,052.30
12	\$907.93	825.44	\$82.49	98,969.81
13	\$774.00	659.80	\$114.20	98,855.61
14	\$774.00	659.04	\$114.96	98,740.64

Debriefing the Lesson

Variable rate mortgages, as we have just seen, require periodic readjustments of the monthly payment to compensate for interest rate changes. In practice, adjustments are normally made every two years, but the size of the changes can be just as dramatic. Lenders are required by law to disclose all of the conditions of the contract, so you need to inquire

before you sign up for one. Even so, interest rates can move higher, or they can be lower, but you can be virtually assured that they will never remain the same over the course of the loan. For this reason, many people prefer fixed rate mortgages to variable rate ones.

a **UNIT RESOURCES:**



Community Resources:

- Invite a loan officer from a local bank or savings and loan to bring some sample amortization schedules to class. Ask for examples of both fixed rate and variable rate mortgages.
- Invite a mortgage lender to class to talk about the differences between fixed rate and variable rate mortgages.



Teacher Resources:

- [A Consumer's Guide to Mortgage Settlement Costs](#) Washington, D.C. Board of Governors of the Federal Reserve System, Washington, D.C. 20551-0001. Short, useful, and free guide for those who need more information on the incidental costs of obtaining a mortgage.
- [A Consumer's Guide to Mortgage Refinancing](#) Washington, D.C. Board of Governors of the Federal Reserve System, Washington, D.C. 20551-0001. Short, useful, and free guide for those who want to replace a high-rate mortgage with a lower-rate one.
- [Opening the Door To a Home of Your Own](#). Useful booklet from the Fannie Mae Foundation discussing topics related to the financing, personal credit history, and the affordability of mortgage payments.



World Wide Web Resources:

<http://loan.yahoo.com/m/living.html/> Yahoo's list of links to mortgage refinancing; other useful articles on home ownership

<http://loan.yahoo.com/m/mortcalc.html> Useful list of real estate, home financing calculators from Yahoo

<http://realestate.yahoo.com/realestate/calculators/amortization.html>
Yahoo's amortization calculator; has features for extra payments

<http://www.eloan.com/cgi-bin/amortcalc> Generate a complete amortization schedule with this Yahoo calculator

<http://www.eloan.com/cgi-bin/eloansimple> Simple mortgage payment calculator; computes amount of monthly payment

<http://www.interest.com/calculators/index.shtml/> Mortgage and other calculators; links to monthly payment, how much you can afford to borrow, discount points, and interest payments calculators

<http://www.ieacle.ie./mortgage/> One of the better mortgage calculators on the web; graphical presentation of interest and principal allocation of monthly payment, accommodates additional payments

<http://www.altavista.com/> Search for “mortgage calculator” on this popular search engine for a list of the latest mortgage calculator sites

<http://www.excite.com/> Search for “mortgage calculator” on this popular search engine for a list of the latest mortgage calculator sites

ACTIVITY SHEET 8-1

COMPUTING INTEREST AND THE REDUCTION OF PRINCIPAL

Directions: The three problems below illustrate the way in which the amount owed on mortgage decreases as periodic monthly (amortized) payments are made. The monthly payment is assumed to be \$733.76, so all you have to do is to compute the amount of interest and new principal owed for three consecutive payments. Remember that the reduction of principal is the residual after interest payments are made.



1. Find the interest on a \$100,000, 8-percent, 30-year mortgage using the simple interest formula when the \$733.76 payments are made monthly:

$$\text{Interest} = (\text{Principal})(\text{Rate})(\text{Time})$$
$$=$$

The new principal amount owed is: \$100,000 - _____ = _____

2. The amount of interest owed when the *second* \$733.76 payment is made is:

$$\text{Interest} = (\text{Principal})(\text{Rate})(\text{Time})$$
$$=$$

The amount of principal owed after the *second* periodic payment is:

$$\text{_____} - \text{_____} = \text{_____}$$

3. The amount of interest owed when the *third* \$733.76 payment is made is:

$$\text{Interest} = (\text{Principal})(\text{Rate})(\text{Time})$$
$$=$$

So, the amount of principal owed after the *third* periodic payment is:

$$\text{_____} - \text{_____} = \text{_____}$$

ACTIVITY SHEET 8-2 THE MORTGAGE AMORTIZATION SCHEDULE

Directions: The conditions for the hypothetical variable rate mortgage below are as follows. (1) .



The Mortgage Amortization Schedule

Payment Number	Monthly Payment	Interest Owed	Principal Reduction	New Principal Owed
1	\$733.76	\$666.66		\$99,932.90
2	733.76		67.54	
3	733.76	665.77	67.99	99,797.37
4	733.76			99,728.92
5	733.76	664.86	68.90	99,660.02
6	733.76		69.36	99,590.66
7	733.76	663.94	69.82	99,520.84
8	733.76	663.47	70.29	
9		663.00	70.76	99,379.80
10	733.76	662.53		99,308.57
11	733.76	662.06	71.70	
12	733.76	661.58	72.18	99,164.68
13	733.76	661.10	72.66	99,092.02
14	733.76			

Note that you will eventually pay the lender \$264,153.60 (or \$733.76 times 360 months.) Since \$100,000 is principal, total interest payments amounted to \$164,153.60!

ACTIVITY SHEET 8-3 REDUCING THE BURDEN – MAKING EXTRA PAYMENTS

Directions: One way to reduce the burden of the mortgage is to make additional payments. For example, rather than make the required monthly payment of \$733.76, suppose that you add an extra \$100 every month to increase the payment to \$833.76.

The partial table below illustrates how the principal amount owed goes down as monthly payments of \$833.76 are made during the first few months of the \$100,000, 8-percent mortgage. Again, your task is to fill in the blanks.



The Mortgage Amortization Schedule

Payment Number	Monthly Payment	Interest Owed	Principal Reduction	New Principal Owed
1	\$833.76	\$666.66		\$99,832.90
2	\$833.76	665.55	\$168.21	
3		664.43	\$169.33	99,495.36
4	\$833.76		\$170.46	99,324.90
5	\$833.76	662.16		99,153.31
6	\$833.76	661.02	\$172.74	
7		659.87	\$173.89	98,806.67
8	\$833.76		\$175.05	98,631.62
9	\$833.76	657.54		98,455.40
10	\$833.76	656.37	\$177.39	
11		655.18	\$178.58	98,099.43
12	\$833.76		\$179.77	97,919.67
13	\$833.76	652.80		97,738.70
14	\$833.76	651.59	\$182.17	

Because of the extra payments, the loan is paid off in 242 months, which is approximately 20, rather than 30, years. In this case you will eventually pay the lender \$201,770 (or \$833.76 times 242 months.) Since \$100,000 is principal, total interest payments amounted to \$101,770—saving you about \$62,384 in interest payments.

ACTIVITY SHEET 8-4 VARIABLE RATE MORTGAGES

Directions: Assume that you have just obtained a \$100,000, 25-year, 9-percent variable rate mortgage. The terms are such that the size of the payment will be recomputed every six months if interest rates change, although rates will never vary more than 6 percentage points over the life of the loan (never get higher than 15 percent, nor lower than 3 percent). Complete the schedule below, assuming that rates increase to 10 percent for the 7th payment, then drop to 8 percent for the 13th payment.



The Mortgage Amortization Schedule

Payment Number	Monthly Payment	Interest Owed	Principal Reduction	New Principal Owed
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				